An Iterative Possibilistic Image Segmentation System: Application to breast cancer detection

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Abstract – A novel approach for digital mammograms segmentation is proposed. This approach aims to segment the mammograms using an iterative fusion process of information obtained from multiple sources of knowledge (contextual, image processing algorithm, a priori knowledge, etc). Initial Fuzzy Membership Maps (IFMMs) of different thematic classes are first estimated using available information. These IFMM's are then interpreted as Possibility Distribution Maps (PDMs), which represent the possibility for each analyzed pixel to be one of the different thematic classes in the considered image, these possibility values are then iteratively updated using contextual (spatial) information. An additional class called "Rejection" is used to manage ambiguity and to delay the segmentation operation until the establishment of high level possibility degrees for these pixels. The segmentation results are given as a thematic map as well as a confidence curve evaluating the segmentation result quality.

Keywords: Mammography, Fuzzy Segmentation, Iterative Fusion, Possibility Theory.

1 Introduction

Mammography is considered as the main investigation tool to detect breast cancer [1-4]. Early tumor detection is the key for improving breast cancer prognosis. This detection relies on locating masses and micro-calcifications that both indicate suspicious regions [5]. On mammograms, tumor regions often correspond to brighter pixels than the surrounding tissues [6]. Simple eye examination of mammograms is often considered as prone to detection errors [2]. For this reason Computer Aided Detection (CAD) techniques have been developed to assist the radiologists in the analysis of the difficult cases, to reduce the number of useless biopsies in the recommendation of biopsies and to confirm diagnosis [2-5]. In spite of advances in medical imaging, some difficulties remain in the analysis of mammograms, mainly due to following points [5]:

- Regions of interest (ROIs) are often extremely small.
- The ROIs of different sizes also have various shapes and "fuzzy" boundaries.
- Pixels intensity difference between ROIs and surrounding tissues can be extremely low.

This last point is moreover amplified by the presence of noise, inherent to most medical images acquisition systems. Furthermore, mammogram histograms are usually not bimodal, thus leading to class overlapping problems [7,8].

The CAD system, proposed in this study, is based on the use of fuzzy sets and possibilistic reasoning, is developed in order to represent and manage such ambiguous situations. The tackled problem is to assign each pixel from the analyzed mammogram to one of the three following classes of interest: Background, Tumor, and Benign Tissue.

The main idea of the proposed approach is to consider the mammogram images segmentation as a fuzzy classification problem and to refine the membership maps (to different thematic classes) in an iterative way. This refinement is conducted by: A. Interpreting the membership values as possibility degrees, B. Applying the maximum possibility decision rule in order to establish "intermediate" segmentation results (i.e. thematic maps), and then, C. Gradually eliminating unclassified pixels, as it has been proposed in [9] (by the integration of multisensor data and contextual information to characterize landscape parcels).

The use of this approach is justified by the fact that mammographic images hold diagnostic information with some natural degrees of fuzziness [5,10]. This is first due to the nature of mammography and breast structure. In fact, gray levels of mammogram pixels correspond to a mixture of several tissues. Such pixels are called "mixels" [11]. Besides, we have to consider the radiologist's analysis and decision as the truth reference for the evaluation of any CAD approach. The radiologist's human reasoning for establishing the diagnostic information is ambiguous due to the inherent fuzziness in human perception [5,10]. That is, a pixel or a region can be partially classified as tumor or benign tissue, depending on several knowledge sources (as, for instance, the surrounding tissues).

The fuzzy set theory is considered as a powerful tool to represent such imprecise and ambiguous information [12]. The major advantage of fuzzy sets on the traditional set theory is the ability to allow the pixels to possess intermediate grades of membership to different thematic classes of the image (Figure 1).

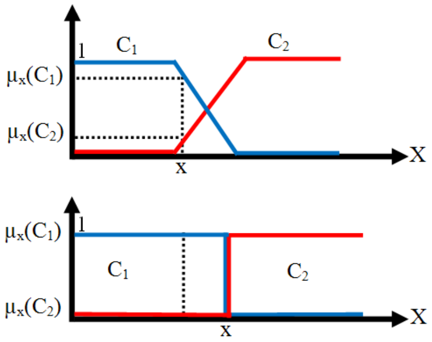


Figure 1. The fuzzy *vs.* hard spaces partition

As already mentioned, the use of possibilistic reasoning mechanism is used through the proposed approach. Several reasons justify the use of this approach:

1. Representing the ambiguous nature of imperfection of the observed pixels;
2. Offering a homogeneous and solid framework for reasoning and for the integration of different knowledge and information sources like the important contextual source of information [13-15];
3. Making intermediate classification decision through the use of different potential decision rules starting by the simple maximum possibility rule to more sophisticated rules integrating decision constraints.

This paper is organized as follows. Section 2 gives a brief review of existing iterative methods used for image segmentation. Section 3 details the proposed segmentation approach. Results concerning the application of the proposed method to a reference mammogram database are given and discussed in section 4. Conclusion and perspective are discussed in Section 5.

2 Overview of mammogram iterative segmentation methods

Many techniques, mostly dedicated to breast cancer detection, can be found in the literature. In this section, we present the major iterative methods in order to compare their concepts and performances with the proposed iterative scheme. The main reason of using iterative approaches is due to their similarity with the human reasoning while analyzing images. A particular attention is made here to methods relying on the use of similar fuzzy concepts.

2.1 Iterative threshold based segmentation

Thresholding is the first image segmentation method [16]. The threshold value is either fixed by the user, or automatically determined by an optimization procedure with respect to given performance criterion. The result of this procedure can be iteratively refined and improved.

In [17], an initial threshold t is defined as the global intensity mean of the mammogram. Then, the mean intensities of pixels beyond and below the threshold t are calculated and denoted respectively by m_u and m_d. The threshold value t is then updated in a recursive manner, with $(m_u + m_d)/2$. This updating procedure is repeated until the “convergence” of the threshold t value (i.e. leading to non significant results modification).

Threshold estimation techniques can be classified into two categories: Global Threshold (GT) estimation [16,18] and Local Threshold (LT) estimation [16].

None of these thresholding techniques are efficient enough to separate masses, neither micro-calcifications, from surrounding tissues [3]. Actually, outputs of both GT and LT are mainly used as a starting point for several other segmentation algorithms, such as Markov random field [3].

2.2 Iterative segmentation using region growing

The basic idea of region growing techniques is to find a set of seed pixels that will be considered as the reference value of the searched region, and then to iteratively integrate surrounding pixels to this ROI according to a similarity criterion based on *e.g.* grey level, texture or color [19,20].

The major asset of this approach is to naturally manage the spatial context of the pixels [3]. Nevertheless, the computation complexity makes this approach difficult to use on images where small regions from different classes are highly interlaced and influenced by noise [3,11].

2.3 Iterative segmentation using edge detection

The edge detection technique depends on the boundary based attributes instead of region based attributes. Various edge detection operators have been proposed such as, Roberts gradient, Sobel, etc. [21]. Some applications have been tested in this direction [22], but results undergo the effects of possible low contrast between searched objects and surrounding tissues, poorly defined and very irregular boundaries [19].

To address this problem, an improved active contour algorithm has been proposed in [23]. This algorithm has been considered as an iterative technique for contour extraction where an initial contour is placed near the expected boundary and the contour converges iteratively to final expected position. Applying of this conventional algorithm is useful to treat such problems in which contrast is low and the contour is ambiguous [23]. Nevertheless, the time and the complexity of the computation are limited to initial contour, because if the initial contour is not initialized near the expected final position, the convergence is not guaranteed, and the computation time becomes expensive.

2.4 Iterative segmentation using Markov fields

Mammography segmentation based on Markov fields has been used to extract the ROIs from the surrounding tissues [24]. This class of methods corresponds to statistical approaches. It aims to estimate a maximum a posteriori Map function and to maximize the posterior probability function of the segmented image Y. This function is expressed as:

$$Y_{MAP} = \arg\max_{Y}(P/(Y = y/X = x) \qquad (1)$$

Where Y, X : denote respectively the original image and the segmented image. In fact, image segmentation based on Markov fields is capable of representing the image spatial context [25], but on the other hand, the time of the maximization process is expensive [3].

2.5 Iterative segmentation based on fuzzy set theory

Due to the particularity of the mammograms, the classic segmentation methods might not lead obtaining an exact result [3]. The Fuzzy set theory pioneered by L. Zadeh [26] has been introduced for mammograms segmentation.
Fuzzy set theory supplies us with a powerful mathematical tool for modeling the human ability to reach conclusions when the information available is imprecise, incomplete and not totally reliable.
Given the universe $\Omega = \{X_1, X_2,.., X_n\}$, a fuzzy set A over Ω is defined as the set of ordered pairs $A = \{(X, \mu_A(X)), X \in \Omega\}$ where $\mu_A(X) \in [0,1]$ denotes the membership function of the element X to the fuzzy set A.
An important difference with the classic (crisp) set theory is that all the elements X of the universe Ω “belong” to the fuzzy set A. Concerning image segmentation approaches, there are basically two kinds of fuzzy iterative methods segmentation which are: the fuzzy threshold segmentation and the fuzzy c-means clustering (FCM) algorithm.

2.5.1 Fuzzy thresholding segmentation

Classical thresholding techniques are effective to segment the objects with clear boundaries. Few methods using fuzzy thresholding are proposed in the literature for solving this problem. These methods are based on a simple optimization of fuzzy measures to find the optimal threshold, such as the fuzzy entropy, the compactness and the fuzzy index [7, 27-31]. In fact, the threshold obtained by a simple optimization of fuzzy measures is not effective, because “it is difficult to determine an accurate membership function for a fuzzy set, because there is an uncertainty in its shape, its location or in its other parameters” [32]. Consequently, it is necessary to refine the membership function and the threshold value.

2.5.2 Fuzzy c-means clustering segmentation

The FCM is based on the use of fuzzy set theory to define the image classes, where all the classes are characterized by their centre of gravity and every pixel has its membership degree to every class [33]. The determination of the different class centers of gravity is conducted through the optimization of a cost function J.
Mathematically, the standard FCM cost function is expressed as:

$$J_M = \sum_{\substack{i=1\\ j=1}}^{\substack{N\\ C}} u_{ij}^{m} \left\| x_i - c_j \right\| : 1 \le m < \infty \qquad (2)$$

m, u_{ij} denote respectively, any real number greater than 1, and the membership of the pixel x_i to the class c_j and ||*|| is any norm expressing the similarity between the measured data and the center. The classification using FCM is iteratively achieved by optimizing the previous cost function. The results of segmentation using FCM prove to be effective to detect simple objects and to be more effective for high resolution images [34]. Furthermore, the FCM method is considered as a semi automatic algorithm because it requires a prior knowledge of the number of classes and the time of the maximization is expensive [34].

The aforementioned methods present some major drawbacks: They are based on statistical features, for complex images, the identified ROIs may be not exactly detected. Furthermore, some methods are complex, expensive time consuming and they could not effectively treat the ambiguity concept existing in mammograms. So, the proposed system takes advantage of these methods to overcome the ambiguity existing in mammograms. Firstly, instead of using the intensity value to represent the pixel, the possibility value is used to better characterize the property of mammograms. Secondly, an iteratively possibilistic knowledge integration and propagation processes are used.

3 Proposed segmentation system

The main target of the segmentation process is to assign each pixel from the analyzed mammogram to a particular class of interest, such as background, tumor, and benign tissue. The image resulting from the pixels labeling is henceforth referred to as a thematic map. An important issue is rarely addressed through segmentation systems. It concerns the fact that the segmentation results lack additional information related to the degree of certainty associated with each thematic class decision. In the proposed system, a pixel X is described as a vector in an N dimensional space:
$X = [x_1, x_2,..., x_n], x_n \in S_n, n = 1,2,..., N$,where S_n denotes the n^{th} knowledge source observation and N is the

number of sources. The universal set S is the Cartesian product representing the multisource observation space. If $C=[C_1,C_2,..,C_M]$ is the set of M image predefined classes, then, based on the fuzzy set theory, each class C_M is defined as a fuzzy set over S with $\mu_x(C_{m,n}), m=1,2,...,M$ denoting the membership degree for which the pixel $X\in S$ may be treated as belonging to the class C_M .

Possibility theory was introduced by Zadeh [15] in 1978, and developed by Dubois and Prade [35] in 1988, it represents a tool allows to represent uncertainty linked with imprecise information in order to enable reasoning on imperfect information by the use of a possibility distribution $\pi(x)$. The possibility distribution $\pi(x)$ of a singleton $x\in\Omega$ is mapping: $\pi:\Omega\rightarrow[0,1]$. The possibility distribution of x shows the possibility value of its occurrence such as: $\pi(x)=1$ means that x is completely possible and $\pi(x)=0$ means that x is completely impossible. To characterize an event A, two measures are also defined: the possibility measure $\prod(A)$ and the necessity measure $N(A)$. The possibility measure is defined as follows:

$$\prod(A)=\max_{x\in A}\pi(x) \qquad (3)$$

This measure shows the degree to which the event A is possible on Ω .

On the other hand, the necessity measure is defined as following:

$$N(A)=1-\Pi(\overline{A})=1-\max_{x\in A}(1-\pi(x)) \qquad (4)$$

Where $\overline{A}$ denotes the complement of A. This measure shows the degree to which A is certain on Ω such as: $N(A)=1$ means that A is completely certain and $N(A)=0$ means that A is completely uncertain.

The proposed segmentation system, depicted in figure 2, is an iterative image segmentation system based on the concept of “possibilistic” knowledge propagation. Assuming the number of thematic classes contained in the analyzed image, is given as a prior knowledge (corresponding to the closed world assumption in classification systems), the proposed system can be decomposed into the following sub-systems:

1. Image preprocessing system;
2. Fuzzy class membership estimation system;
3. Fuzzy membership fusion system;
4. Possibilistic decision system;
5. Iterative possibilistic knowledge propagation system.

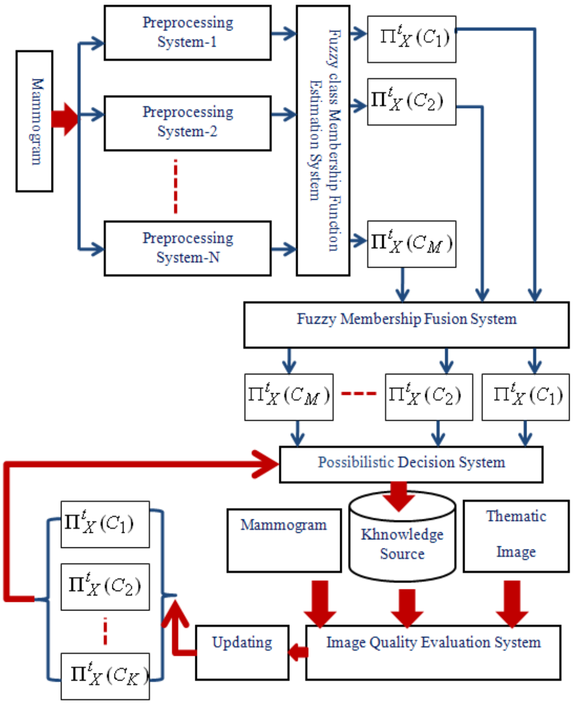


Figure 2. The proposed system

3.1 Image pre-processing system

The image preprocessing system constitutes the first step at the pixel level. It presents an additional knowledge source which aims to provide different texture information of mammograms and to extract informative elements from the original image. This system may include simple filtering or edge detection, etc. In our system, two knowledge sources may be used: the Wiener filter and the 3x3 mean filter scanning the original image. The use of these filters intends to achieve an image smoothing operation. The Wiener filter produces better results than the mean filter in terms of preserving the edges and other high frequency parts [36]. Notice that, the term "source" is used in this study in order to indicate different information or provided data. This data may be the original data as well as a data issued from processing image algorithms or any new features computed over them.

3.2 Fuzzy class membership estimation system

The aim of the fuzzy membership estimation system is to exploit the output of the preprocessing system and to determine the membership degree $\mu_x(C_{m,n})$ of each pixel x , to each thematic class C_M , over the n^{th} source universe S_n. The estimation process can be based on experimental knowledge or on the use of some algorithms dedicated to optimize an objective function [9].

Among the proposed estimation methods, [37] considers the probability distribution as a possibility distribution or as the membership function [38]. Other proposed methods, are based on the optimization of some objectives functions related to the image properties or based on fuzzy measures, such as, the fuzzy entropy [5,7,27,31], the compactness [39] and the fuzzy index. Hence, the estimated membership function and its parameters using these methods generally depend on the parameters of the image.

We propose to use here an estimation process based on the maximization of the fuzzy entropy [40]. The optimization is carried out using the Simulated Annealing Algorithm [41]. The estimated membership function represents "*a priori*" information that can be considered as the expression of a fuzzy proposition, in relation with the event "*x* belongs to class "C_m". Moreover, such a fuzzy proposition induces a possibility distribution [13] that characterizes the realization of the above event, with $\pi_x(C_{m,n}) = \mu_x(C_{m,n})$.

3.3 Fuzzy membership fusion system

The fusion process aims at combining pieces of information, each one suffering from some imperfections that hinders the decision process, to obtain information of better quality that allows taking a better decision. So, the theory of possibility allows managing information from different sources, even of heterogeneous natures, and offers numerous fusion operators, generally classified in four categories [14,42], with respects to the behavior when processing *e.g.* pieces of information in conflict: 1) conjunctive operators, 2) disjunctive operators, 3) average operators, and 4) tradeoff operators. In this study, a disjunctive operator $F(\cdot)$ is applied to possibility distributions associated with the different knowledge sources during the estimation stage, to provide an unique distribution for each thematic class:

$$\Pi_x(C_m) = F\big(\pi_x(C_{m,n})\big), n = 1..N \tag{5}$$

The interest of such operator is: its indulgent behavior that is the ability to preserve high possibility degrees. So, the information about all possible classes of a pixel is kept unchanged until next processing stage, where more complex information (spatial context of the pixel) is available to refine the decision (*cf.* section 3.5).

3.4 Possibilistic decision system

A decision rule has to be applied to the fused possibility degrees associated with each pixel to determine its thematic class. Let $\xi_x(C_m)$ be the possibility value associated with site *X* (contextual possibility value). The possibility degree from equation (5) thus becomes:

$$\xi_X^t(C_m) = \Pi_x^t(C_m) \tag{6}$$

Where pixel *X* is characterized by a feature value *x* (for simplification purpose, the contextual possibility value will be referred by its general term $\xi_x^t(C_m)$ in the following). Superscript *t* represents the temporal evolution of possibility degrees in the iterative updating process (iteration index).

At *t*=0, $\xi_x^{t_0}(C_m) = \Pi_x(C_m)$, where $\Pi_x(C_m)$ is the fused possibility measure. Based on this possibility degree, the decision rule used for the initial pixel assignment is defined as follows: *x* is assigned to class C_{m_0} if:

$$\begin{cases} \xi_x^t(C_{m_0}) > \xi_x^t(C_m), m \neq m_0, \text{and} \\ \xi_x^t(C_m) \approx 0, m \neq m_0, \text{and} \\ \xi_x^t(C_{m_0}) \geq 0.9 \end{cases} \tag{7}$$

When a pixel *X* of value *x* verifies these conditions, its membership to class C_{m_0} is reinforced by setting the possibility value $\xi_x(C_{m_0})$ to 1 and its possibility value to all other classes to 0.

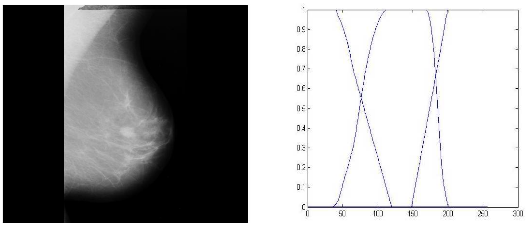
A problem arises for pixels which have close possibility values to different classes. In this case, the concerned site cannot be assigned with certainty to a given class due to the partial nature of membership degrees associated to "mixels". To solve this problem, we define an additional class, called "rejection", corresponding to pixels for which the decision is postponed. The output of the decision system is thus a thematic image containing $K = M + 1$ classes.

3.5 Iterative possibilistic knowledge propagation system

The proposed system integrates the spatial context of image pixels (neighborhood) into the decision process in order to improve the segmentation result (decision concerning "mixels", *etc.*) iteratively. Considering PDMs maps with K classes, the possibility updating system relies on the following general principle: facing a difficult decision (*e.g.* classifying a "mixel"), a human expert would examine homogeneous regions surrounding the considered pixel. The principle of the possibility updating system is to model this reasoning process by considering possibility degrees in a 3×3 neighborhood around the pixel. Actually, these local neighborhoods are used to update possibility values associated with the central pixel. The possibility of a pixel to belong to class C_m is processed as follows:

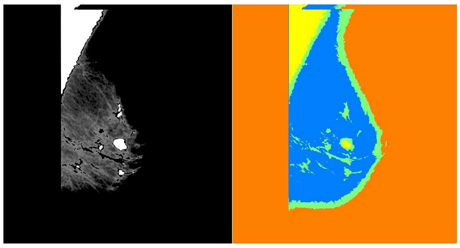
$$\xi_x^{t+1}(C_m) = \frac{1}{9}\sum_{j=1}^{9} \xi_{x_j}^t(C_m) \tag{8}$$

where x_j are pixels features in the neighborhood of the considered pixel. Consequently, a validated decision concerning a given pixel is iteratively propagated to its surrounding region. Another consequence is that the robustness of the decision-making process is also improved by integrating a new knowledge source related to ROI homogeneity. For t >0, the decision rule is the same than equation (7), with a threshold of 0.75 instead of 0.9 to ensure a quick convergence of the algorithm. Moreover, since possibility degrees are iteratively averaged in a given neighborhood, pixels initially belonging to other classes in this neighborhood will not disturb significantly the decision making process.



(a) (b)

Figure 3. (a) Image mdb010 from the test image base and (b) Related possibility distribution



(a) (b)

🟧 Background class

🟦 Benign tissue class

🟨 Tumor class

🟩 Rejection class

Figure 4. (a) Possibility map of the tumor class and (b) The thematic map at t=0

3.6 Image quality evaluation system

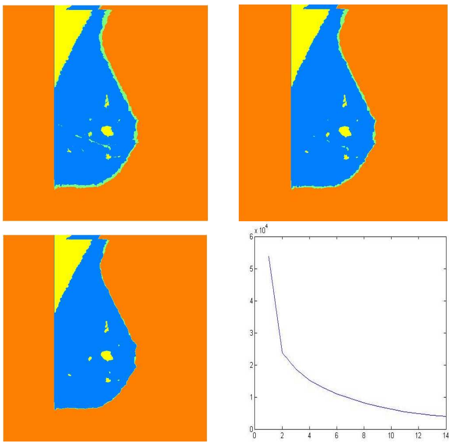
In addition to the thematic image, we intend to provide an indication about the quality of the decision. We use the percentage of non-classified "mixels" as an evaluation of the segmentation quality. Furthermore, the evolution of this quality along the updating process is also used as a convergence criterion to stop the algorithm (Fig.5.d). At the end of the iterative process, possibility values are 0 or 1, and remaining "mixels" are affected to the major class in their neighborhood.

4 Results and discussion

The proposed system was applied to the image base supplied by the "Mammographic Image Analysis Society" (MIAS) [43]. Mammograms are considered to contain three thematic classes: C1: background, C2: lesions, C3: benign tissues.

Figure 3.b shows possibility distributions obtained for an example image from the base. Based on these possibility distributions and the decision rule detailed by equation (7), figure 4.b shows the segmentation result at t=0. On this image, about 10% of pixels are considered as "mixels" (labeled in green). This result confirms the fuzzy nature of mammograms and the difficulty to classify the "mixels" when the decision is taken from distributions only. Intermediate results obtained during the iterative updating process are shown on figure 5.

The convergence threshold was fixed to 0.03% of non-labeled "mixels". With the example of image mdb010, 14 iterations of the updating process were sufficient to satisfy this condition (Fig.5.d).



(a) t=2

(b) t=5

(c) t=11

(d)

Figure 5. (a, b, c) Different thematic images from the image mdb010 during the updating process, and (d) the quality evaluation curve

Iterative updating of possibility values using the spatial context makes regions to be more and more homogeneous. The final thematic maps corresponding to several images from the database (including image mdb010) are shown on figure 6.

For a first qualitative evaluation of the system performances, the process has been applied to four images (Fig.6), and results have been compared with those obtained in [20]. Results show that the proposed system is more robust for the detection of suspicious regions. For instance, the system proposed by Zhang *et al.* failed to detect malignant masses integrated into fatty tissues on images mdb141 and mdb010 (Fig. 6.a and 6.c), as described by object coordinates provided with the test database. Thanks to the management of the spatial context of these ambiguous objects, both of them were successfully segmented by the system described in this paper.

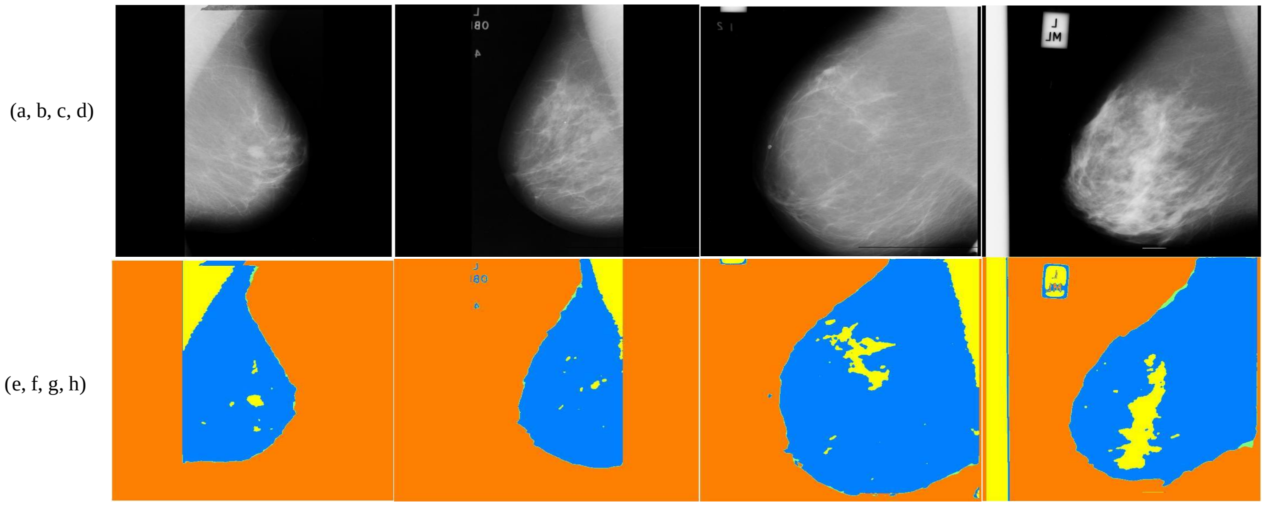


Figure 6. (a) Image mdb010, (b) Image mdb091, (c) Image mdb141,(d) Image mdb145 and (e-h) Related final thematic Images

5 Conclusions and perspectives

In this study, we have proposed a mammogram segmentation system which aims at detecting breast tumors. The proposed system is based on an iterative propagation of possibilistic knowledge. The advantages offered by this system are:

1. Possibility values of the “mixels” are computed in the iterative contextual classification phase and not during the initial decision phase.
2. The proposed system allows modeling the concepts of ambiguity and “hesitation” when some pixels can belong to several classes and not to a single one.
3. The iterative updating of possibilistic values based on the contextual knowledge provides a powerful tool for the knowledge propagation, in order to obtain the final thematic map.

Obtained results show that the proposed system is an efficient second mammogram reader and can detect automatically the suspicious zones without human intervention.

In terms of perspectives, it is clearly shown that the possibility measure provides a tool that allows obtaining a conclusion from available information and to deal with the uncertainty existing in pixel classification. Furthermore, the possibility theory offers another measure, the necessity measure, to handle the uncertainty. Hence, to complete the information about the actual class of a pixel, the measure of necessity will be introduced into the decision stage of the proposed system.

The qualitative analysis proposed in this paper should of course be supported by a quantitative evaluation of the system performances, furthermore considering the influence of tumor size. Detection probabilities can be estimated building the matrix of false positive (FP), false negative (FN), true positive (TP), and true negative (TN) cases, as defined in table 1, either in random ROIs of the image for a technical analysis, or on the whole image for a detection of healthy/pathological patients.

TP	Masses present and detected by the system
FN	Masses present but not detected
TN	Absence of masses confirmed by the system
FP	The system detects masses even if no one is present

Table 1 .Definition of detection cases

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